Acceptance sampling

variables
attributes

Acceptance sampling

Decision on accepting or rejecting a lot

- lot from a supplier
- lot from another department
- lot to be delivered

It is not parameter estimation but hypothesis testing.

Its influence on the production process is indirect only.

The lot is considered as composed of separate items
(e.g. 1000 bearings)

- accepting without checking
- full inspection
- sampling inspection

The full inspection is not error-free either.

The sampling inspection is justified

- if the test is destructive;
- if the full inspection would be too expensive or woud cause delay in production;
- if the error rate of 100% inspection is high enough to make the risk of accepting elements than it would be in sampling inspection;
- if the quality history of the supplier is good, thus the former 100% inspection is to be relaxed, but totally neglecting the inspection is not an option;
- in case of heavy responsibility (medical and healthcare products), thus care is required.

Advantages of sampling inspection as compared with 100% inspection:

- usually less expensive as fewer items are checked
- risk of damage is lower;
- requires less effort
- frequently the error rate of inspection is lower;
- rejecting a whole lot (instead of returning the selected wrong items) makes pressure to the supplier in order to improve the quality.

Disadvantages of sampling inspection as compared with 100% inspection:

- there is a chance for rejecting a good lot or accepting a bad one;
- less information is obtained on the statistical properties of the product or the process;
- requires planning and careful documentation.

It is hypothesis testing: an assumption on a population is either accepted or rejected based on the sample.

The null hypothesis refers on a parameter of the lot, e.g. to the p proportion of defectives.

Error of first kind: A conforming lot is rejected (producer's risk).

Error of second kind: A nonconforming lot is accepted (consumer's risk).

Example 1

The lot size is N=1000, with 1% nonconforming allowed (p=0.01).
A sample of size 80 is taken from the lot.

The acceptability criterion: The lot is accepted if the number of nonconforming items is less or equal to 2 (acceptance limit or acceptance number, *c* or *Ac*), it is rejected if the number of nonconforming items is larger (rejection limit or rejection number, *r* or *Re*).

What is the probability of rejecting a lot when p=0.01 (error of first kind, α)?

k	p=0.01	
	$P(D=k)$	$F(k)=$ $=P(D\le k)$
0	$\binom{80}{0}\cdot 0.01^0\cdot 0.99^{80}=0.4475$	0.4475
1	$\binom{80}{1}\cdot 0.01^1\cdot 0.99^{79}=0.3616$	0.8091
2	$\binom{80}{2}\cdot 0.01^2\cdot 0.99^{78}=0.1443$	0.9534
3	$\binom{80}{3}\cdot 0.01^3\cdot 0.99^{77}=0.0379$	0.9913
4	$\binom{80}{4}\cdot 0.01^4\cdot 0.99^{76}=0.0074$	0.9987

$$\alpha = P\left(D \ge Re \,\middle|\, p_0\right)$$

$$P\left(D \ge 3\right) = 1 - P\left(D \le 2\right) =$$
$$= 1 - F\left(2\right) = 0.0466$$

What is the probability of accepting a lot in which p=0.05 (error of second kind, β)?

k	p=0.05	
	$P(D=k)$	$F(k)=$ $=P(D\le k)$
0	$\binom{80}{0}\cdot 0.05^{0}\cdot 0.95^{80}=0.01652$	0.01652
1	$\binom{80}{1}\cdot 0.05^{1}\cdot 0.95^{79}=0.06954$	0.08606
2	$\binom{80}{2}\cdot 0.05^{2}\cdot 0.95^{78}=0.14457$	0.23063
3	$\binom{80}{3}\cdot 0.05^{3}\cdot 0.95^{77}=0.19783$	0.42846
4	$\binom{80}{4}\cdot 0.05^{4}\cdot 0.95^{76}=0.20043$	0.62889

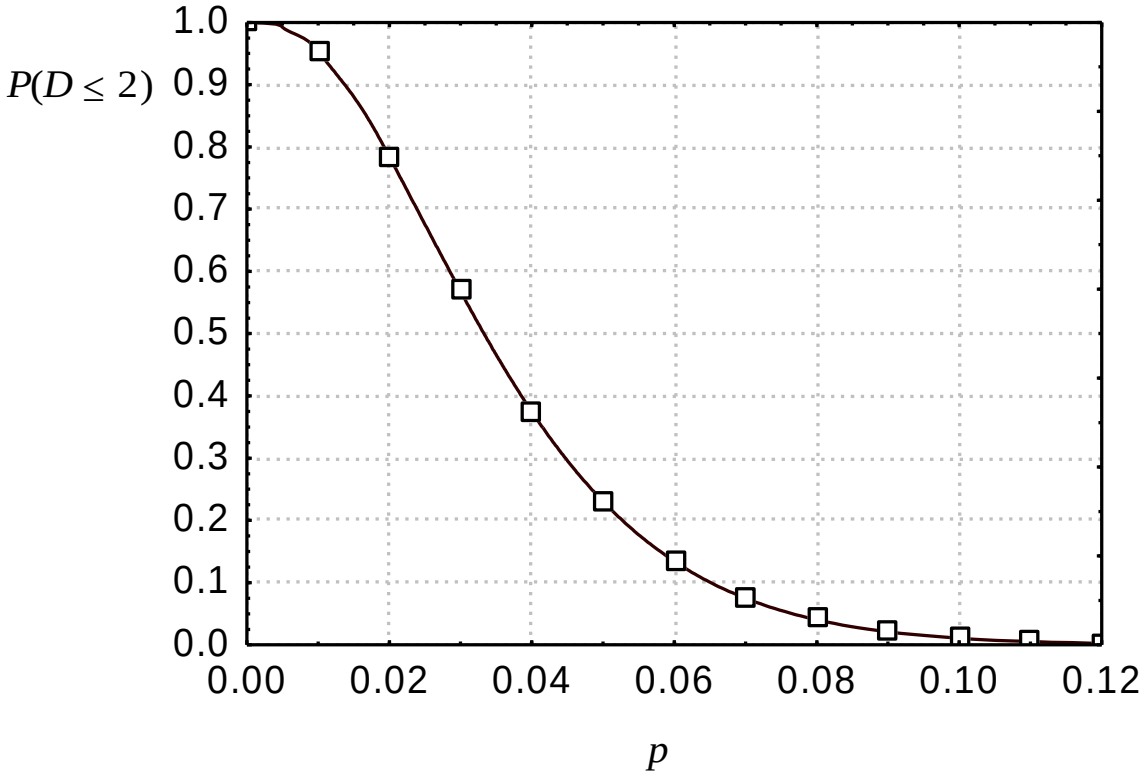
$$\beta = P\left(D \le Ac \middle| p_1\right)$$

$$P\left(D \le 2 \middle| p = 0.05\right) =$$
$$= F\left(2\right) = 0.2306$$

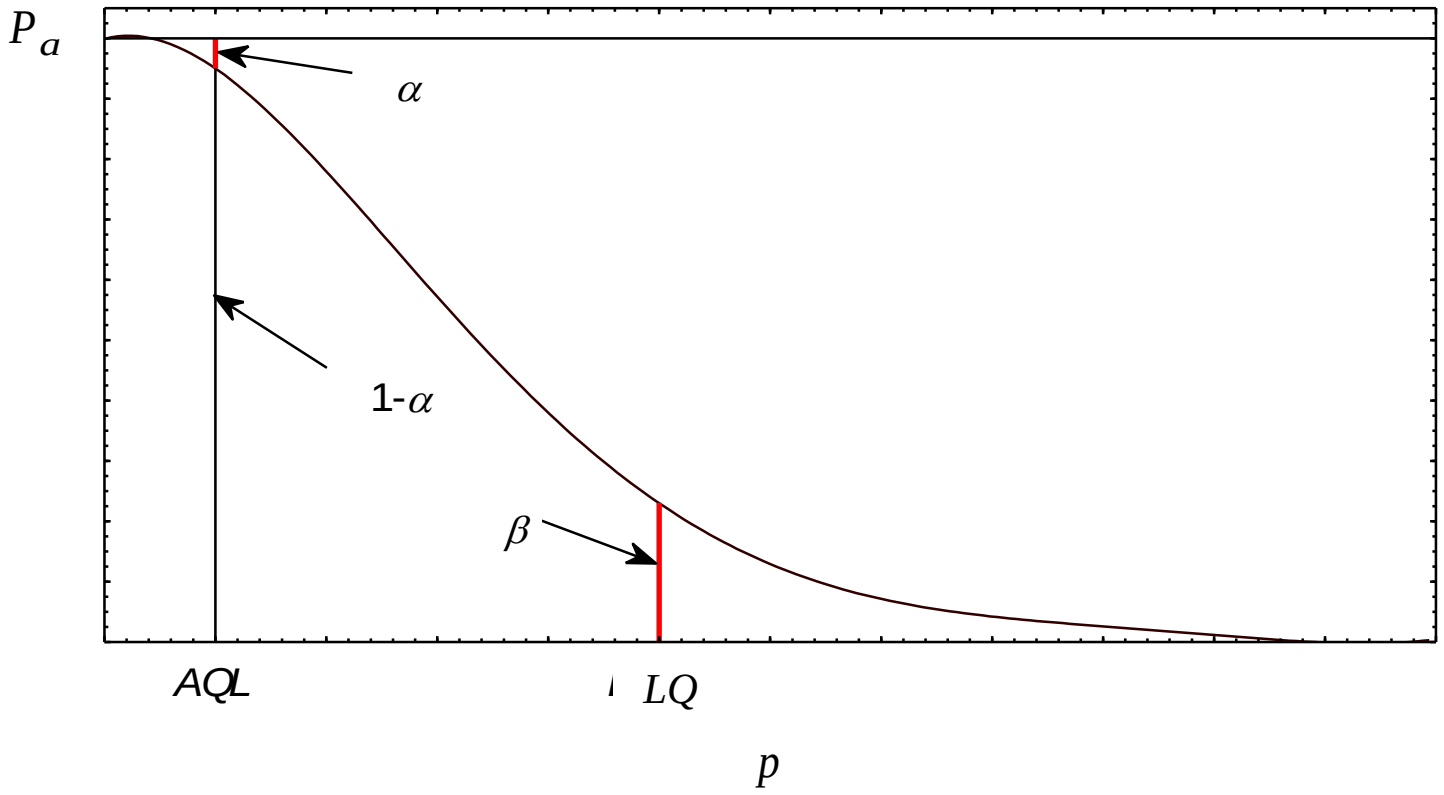
D: Defective

Calculate the P_a probability of accepting a lot with different p (proportion of nonconforming) keeping c=2, and plot the operating characteristic curve!

p	$P_a = P(D \leq 2)$
0.00	1.00000
.01	.95345
.02	.78442
.03	.56812
.04	.37497
.05	.23062
.06	.13445
.07	.07503
.08	.04038
.09	.02106
.10	.01068
.11	.00529
.12	.00256



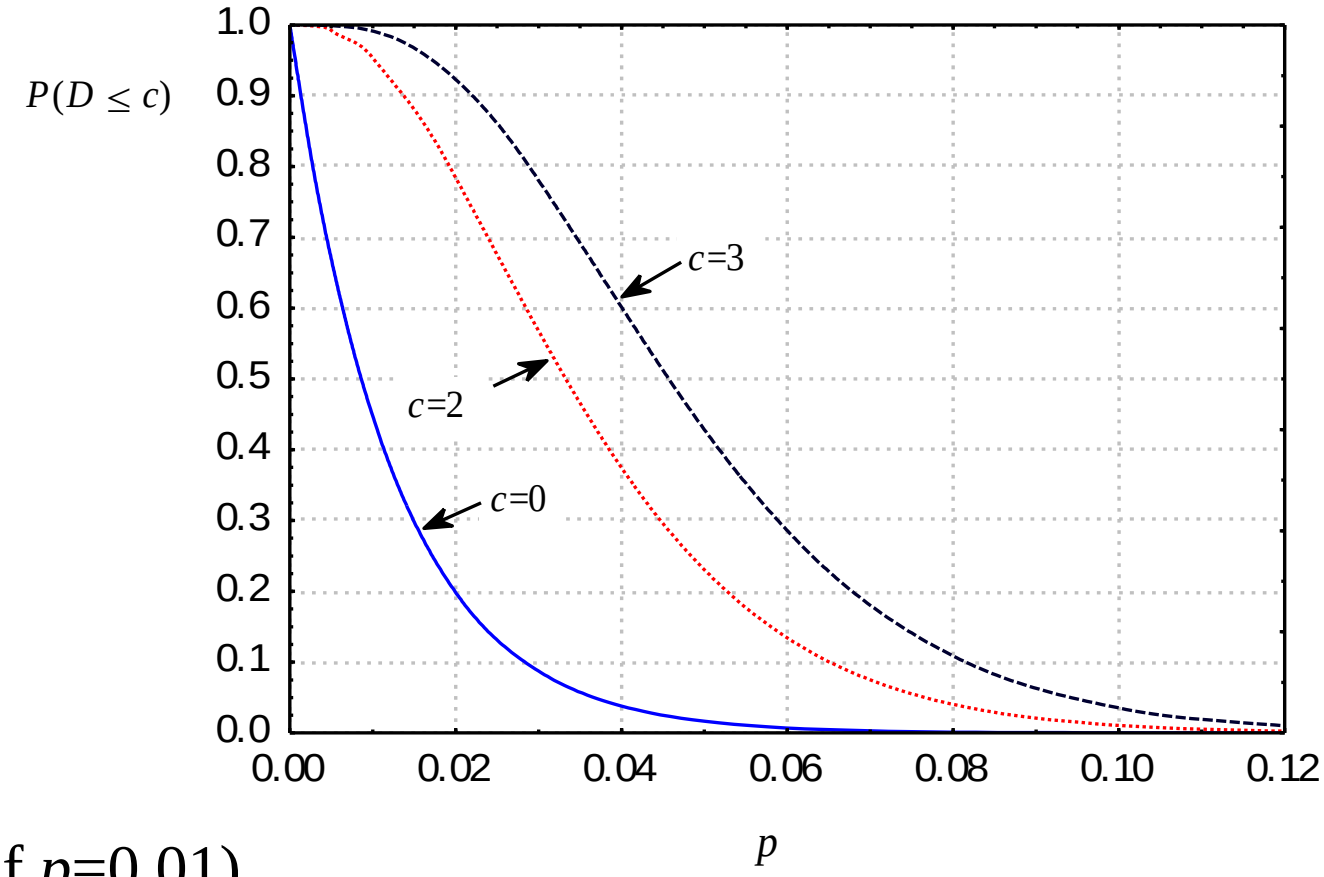
Operating characteristic curve



Acceptable Quality Level AQL=1%,
Limiting Quality LQ=5%

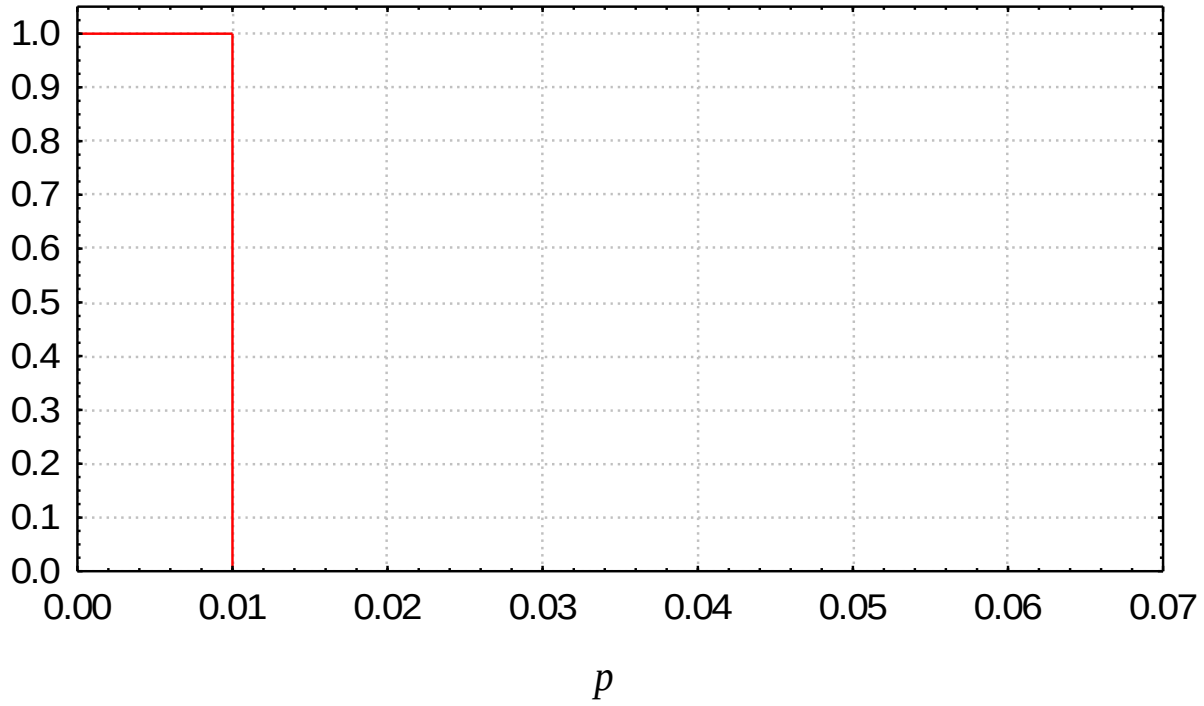
The operating characteristic curve depends bot on n sample size and on c acceptance number.

c=0 not advantageous



(if p=0.01)

In case of full inspection ($n=N$), $\alpha=0$, $\beta=0$



Sampling plan: (N), p_0, p_1, n, $c \rightarrow \alpha$, β

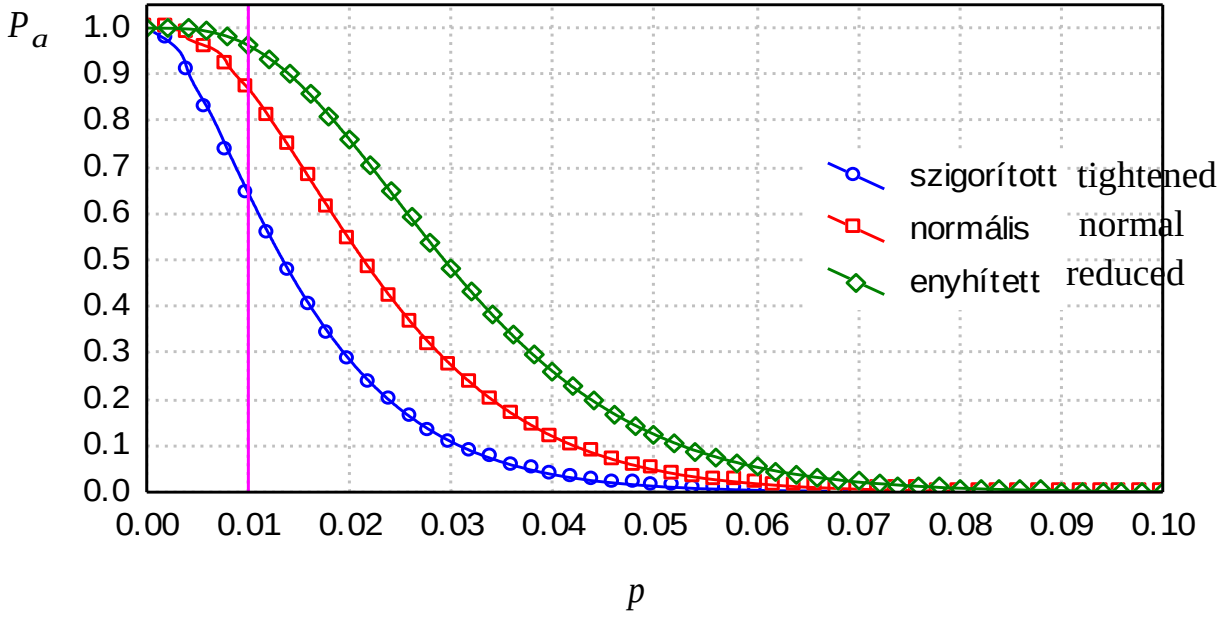
might be (N), p_0, p_1, α, $\beta \rightarrow n$, c two-point

Example

AQL=1% (p_0=0.01), α=0.05,
LQ=5% (p_1=0.05), β=0.1 n=53, c=1.696

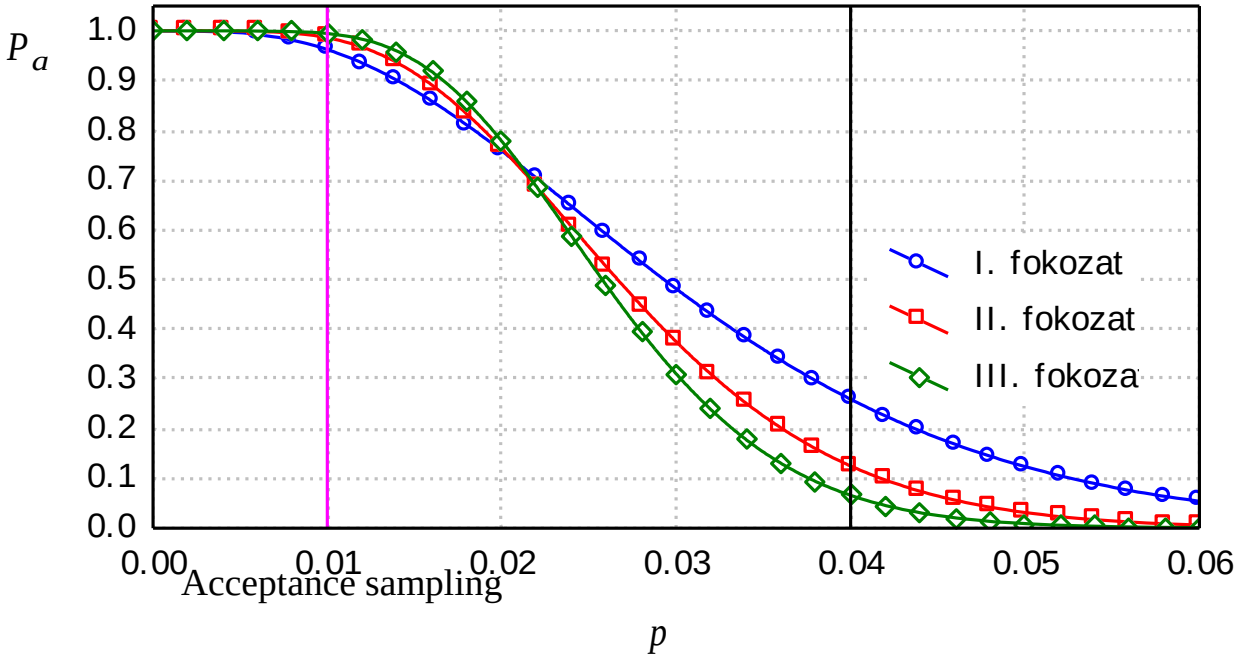
Rounding upwords(c=2),
at $p=p_0$=0.01 P_a=0.984 (α=0.016 instead of 0.05),
at $p=p_1$=0.05 P_a=0.502 (β=0.502 instead of 0.1).

Second (major) difficulty: how to set α and β
standard: single-point



severity

inspection level



Acceptance sampling

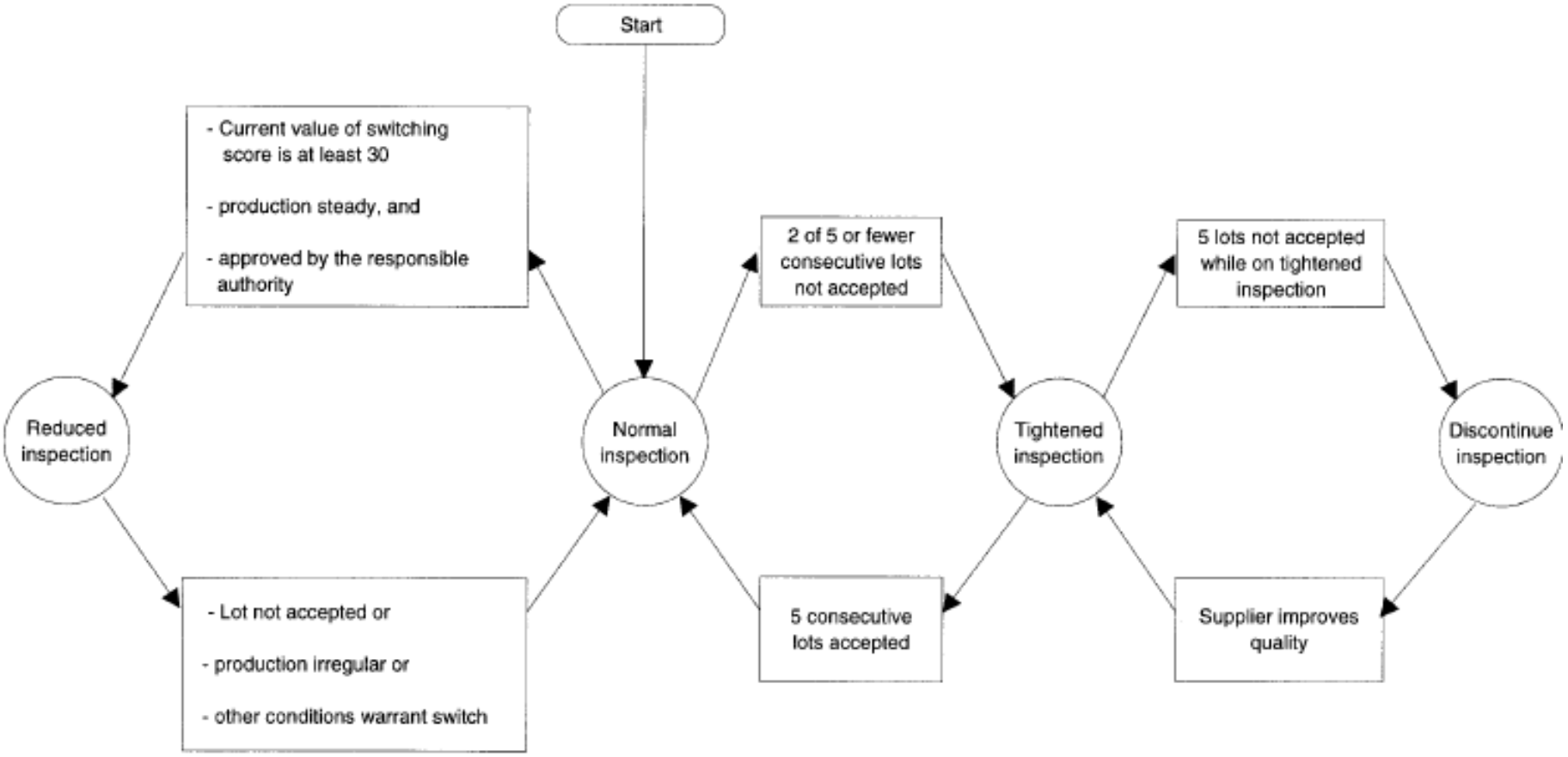
Example 2

A lot of size 1500 is submitted for inspection
ISO 2859-1 (MIL-STD-105D)
N (severity is normal),
II (inspection level),
single sampling plan,
AQL = 1.0%.

Sample size code letters for ISO 2859-1 (MIL-STD-105D)

Lot size		Special				General		
N		inspection level						
		S-1	S-2	S-3	S-4	I	II	III
2-től	8-ig	A	A	A	A	A	A	B
9	15	A	A	A	A	A	B	C
16	25	A	A	B	B	B	C	D
26	50	A	B	B	C	C	D	E
51	90	B	B	C	C	C	E	F
91	150	B	B	C	D	D	F	G
151	280	B	C	D	E	E	G	H
281	500	B	C	D	E	F	H	J
501	1 200	C	C	E	F	G	J	K
1 201	3 200	C	D	E	G	H	K	L
3 201	10 000	C	D	F	G	J	L	M
10 001	35 000	C	D	F	H	K	M	N
35 001	150 000	D	E	G	J	L	N	P
150 001	500 000	D	E	G	J	M	P	Q

sample size code letter


Start
- Current value of switching score is at least 30
- production steady, and
- approved by the responsible authority
Reduced inspection
- Lot not accepted or
- production irregular or
- other conditions warrant switch
Normal inspection
2 of 5 or fewer consecutive lots not accepted
5 consecutive lots accepted
Tightened inspection
5 lots not accepted while on tightened inspection
Supplier improves quality
Discontinue inspection

Switching rules

9.3.3.2 Switching score

The calculation of the switching score shall be initiated at the start of normal inspection unless otherwise specified by the responsible authority.

The switching score shall be set at zero at the start and updated following the inspection of each subsequent lot on original normal inspection.

a) Single sampling plans:

   1) when the acceptance number is 2 or more, add 3 to the switching score if the lot would have been accepted if the AQL had been one step tighter; otherwise reset the switching score to zero;

   2) when the acceptance number is 0 or 1, add 2 to the switching score if the lot is accepted; otherwise reset the switching score to zero.

b) Double and multiple sampling plans:

   1) when a double sampling plan is used, add 3 to the switching score if the lot is accepted after the first sample; otherwise reset the switching score to zero;

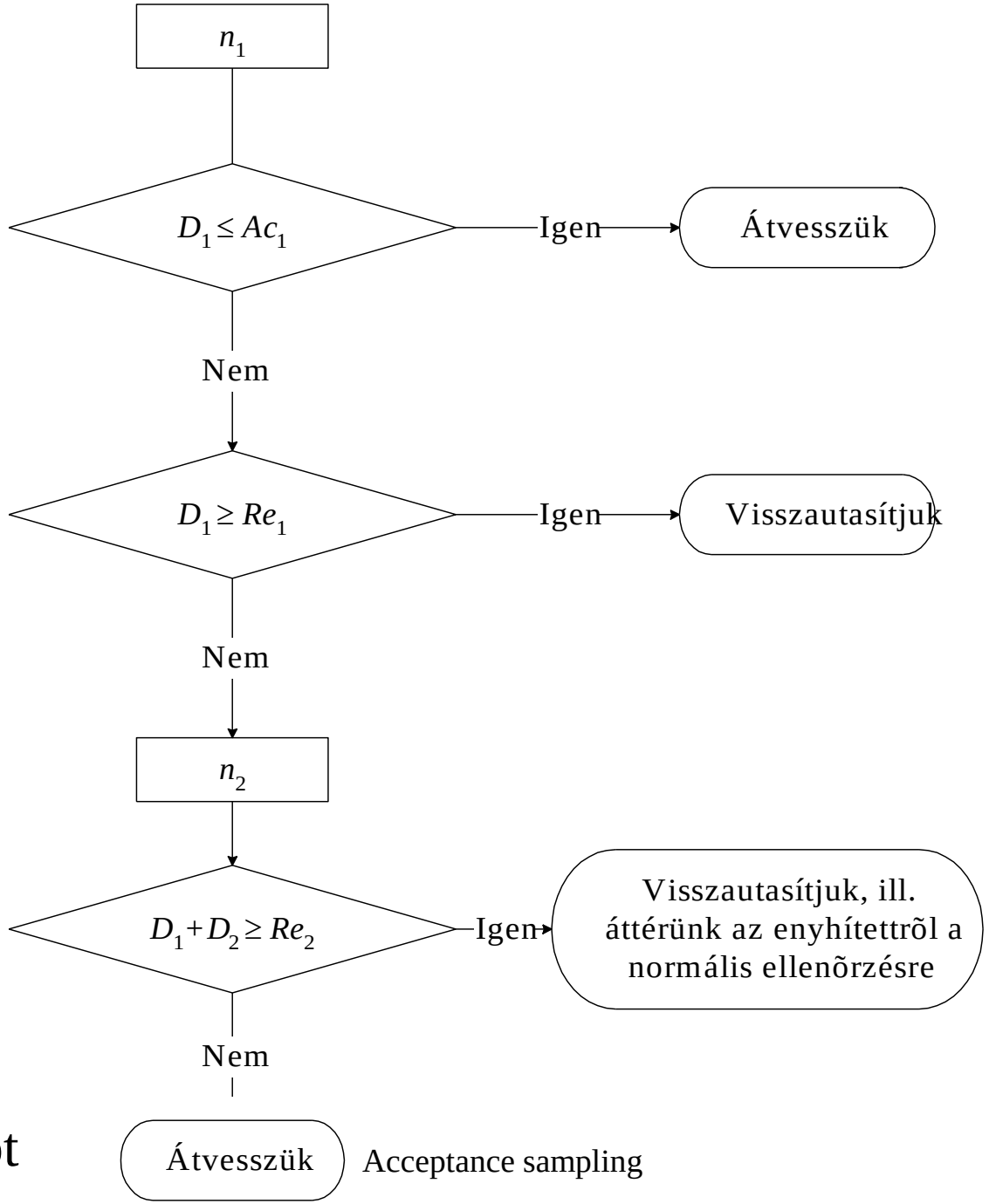
   2) when a multiple sampling plan is used, add 3 to the switching score if the lot is accepted by the third sample; otherwise reset the switching score to zero.

Double and multiple sampling plans (if $p<<p_0$ or $p>>p_0$)

n_1, sample size for the first sample;
c_1, (Ac_1) acceptance limit for the first sample;
r_1, (Re_1) rejectance limit for the first sample;
n_2, sample size for the second sample;
c_2, (Ac_2) acceptance limit for the second sample;
r_2, (Re_2) rejectance limit for the second sample;

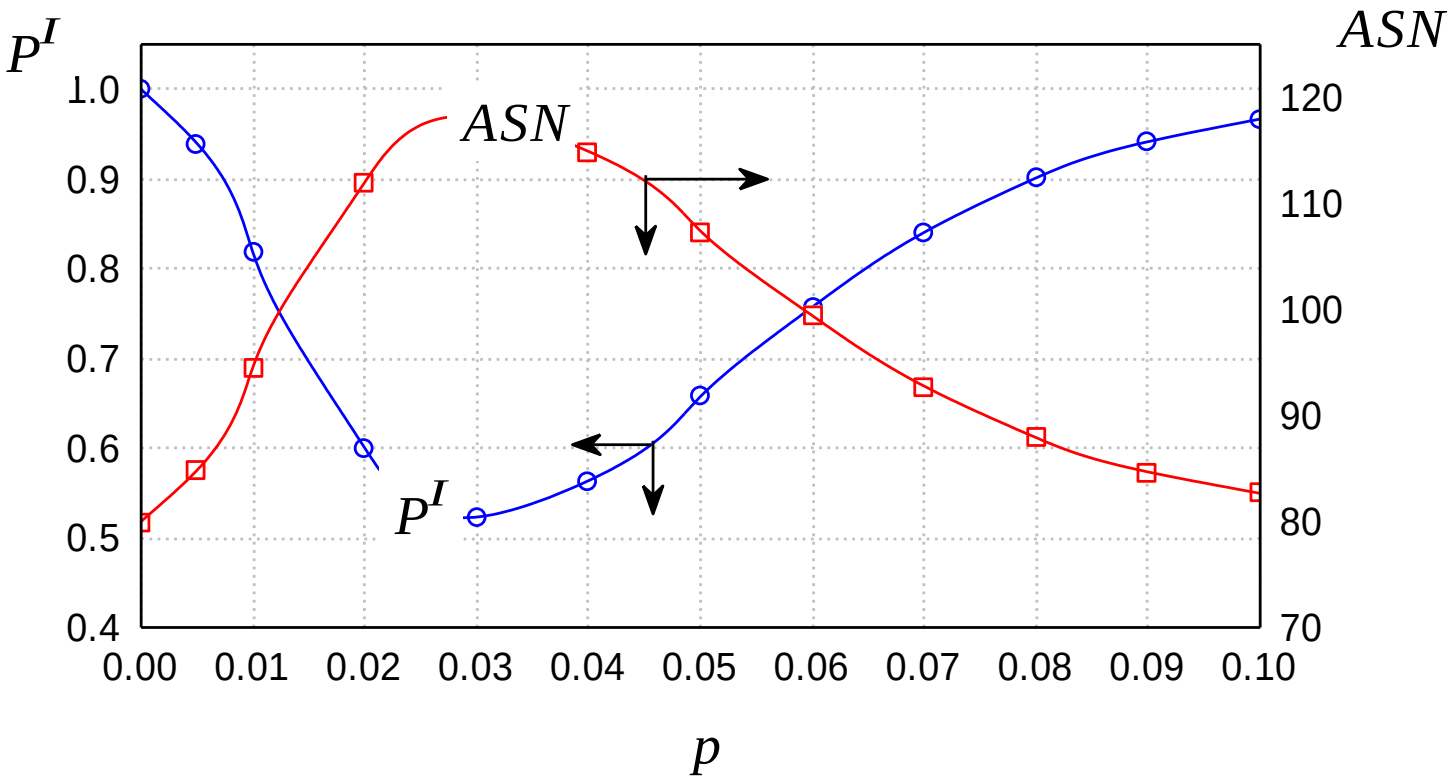
D_1, nonconforming items found in the first sample;

D_2, nonconforming items found in the second sample.


n_1
$D_1 \le Ac_1$
Igen
Átvesszük
accept
Nem
$D_1 \ge Re_1$
Igen
Visszautasítjuk
reject
Nem
n_2
$D_1 + D_2 \ge Re_2$
Igen
Visszautasítjuk, ill. áttérünk az enyhítettrõl a normális ellenõrzésre
reject or return from reduced to normal
Nem
accept
Átvesszük

Average sample size for double sampling

$n_1=n_2=80$ $Ac_1=1$, $Re_1=A$ $c_2=4$ $p_0=0.01$



LQ (Limiting Quality) (Consumer's risk quality)

Used in standard ISO 2859-2 (single lots as compared with series of lots from the same production).
Lots of this (or worse) quality are to be rejected at high probability.
The proportion of nonconforming items in the lot is typically ¼ of LQ.

The ISO 2859 system

(Sampling procedures for inspection by attributes)

2859-0 Introduction to the ISO 2859 attribute sampling system

2859-1 Sampling schemes indexed by acceptance quality limit (AQL) for lot-by-lot inspection

2859-2 Sampling plans indexed by limiting quality (LQ) for isolated lot inspection

2859-3 Skip-lot sampling procedures

2859-4 Procedures for assessment of declared quality levels

2859-5 System of sampling plans indexed by acceptable quality limit (AQL) for lot-by-lot inspection

ISO/TR 8550 Guide to the selection of an acceptance sampling system, scheme or plan for inspection of discrete items in lots

ISO 3951 Sampling procedures and charts for inspection by variables for percent nonconforming

sampling plan: *n*, Ac, Re

sampling scheme: sampling plan+switching rules

sampling system: a collection of sampling schemes (2859-1, -2, -3, 8422)
(2859-0, p. 25)

Theoretical questions for the realization

Assumption of statistical theory: independence.
If it is not fulfilled nothing is valid!

- Lot: manufactured in the same way and circumstances
- AQL acceptability limit, not the requested proportion of nonconforming
- nonconformity and nonconforming item

Lot and lot size

One statistical lot, items manufactured at essentially identical circumstances, as much as possible.

Size of the lot depends on the manufacturing process.

Larger lots are more advantageous (see sample size code letters), require smaller proportion of samples.

If two or more lots are mixed the homogeneity is not assured. If difference is assumed, do not mix the lots.

Lot-by-lot (sequence of lots) from the same supplier

Decision is made for the actual lots but sequence of lots is assumed. This makes the switching rules applicable (accumulation of experience).

At least 10 lots are required for using switching rules.

ISO 2859-2 is used for individual lots.

Sampling

Random sampling: all items have equal chance for being in the sample. Difficult to realize.
Stratified sampling may be an option.

AQL (Acceptance Quality Limit)

If the standard is used to select sampling plans AQL or better lots are accepted at high probability.

AQL does not mean that this proportion is required or really acceptable. If proportion of nonconforming is below AQL, the chance for acceptance is higher.

AQL is the requirement of the customer but it should be realistic for the supplier as well.

Nonconforming item and nonconformity

There may be several nonconformities on a single item.

% if nonconforming and number of nonconformities for 100 items

severe and less severe nonconformity

critical nonconformity